ITEP-TH-31/99
LU-ITP 1999/022
15 December, 1999

EMBEDDED VORTICES AND THEIR INTERACTIONS AT ELECTROWEAK CROSSOVER*

M. N. CHERNODUB
Institute of Theoretical and Experimental Physics
B. Cheremushkinskaja, 25, Moscow, 117259, Russia

E.–M. ILGENFRITZ
Institute of Theoretical Physics,
University of Kanazawa, Kanazawa 920-1192, Japan

AND

A. SCHILLER
Institut für Theoretische Physik
Universität Leipzig, D-04109 Leipzig, Germany

Abstract. We study properties of Z–vortices in the crossover region of the $3D$ $SU(2)$ Higgs model. Correlators of the vortex currents with gauge field energy and Higgs field squared ("quantum vortex profile") reveal a structure that can be compared with a classical vortex. We define a core size and a penetration depth from the vortex profile. Z–vortices are found to interact with each other analogously to Abrikosov vortices in a type–I superconductor.

* Presented by the first author at the workshop "Lattice fermions and structure of the vacuum", 5-9 Oct. 1999, Dubna, Russia.

1. Introduction

Although the standard model does not possess *topologically stable* monopole– and vortex–like defects, one can define so-called *embedded* topological defects [1, 2]: Nambu monopoles [3] and Z–vortex strings [3, 4]. In our numerical simulations of the electroweak theory [5] we have found that the vortices undergo a percolation transition which, when there exists a discontinuous phase transition at small Higgs masses, accompanies the latter. The percolation transition persists at realistic (large) Higgs mass [6] when the electroweak theory, instead of a transition, possesses a smooth crossover around some "crossover temperature" (see Refs. [7]).

We worked in the $3D$ formulation of the $SU(2)$ Higgs model. This report is restricted to results obtained in the crossover regime (assuming a Higgs

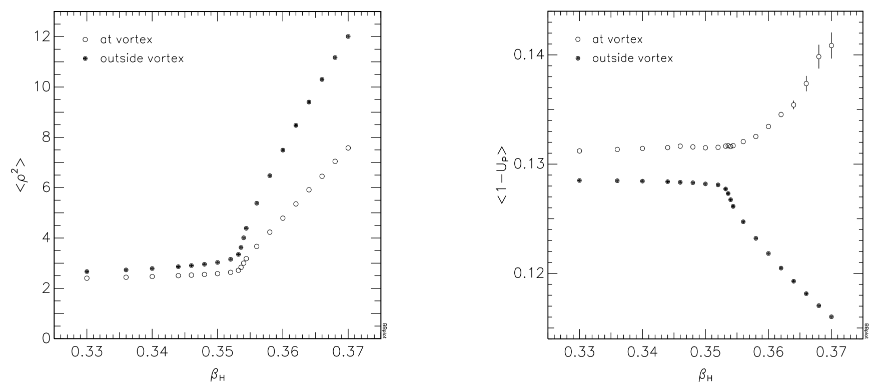


Figure 1. Higgs modulus squared and gauge field energy inside and outside of a vortex *vs.* β_H, $\beta_G = 8$.

boson mass $\approx$ 100 GeV). Details of the lattice model can be found in [8]. The defect operators on the lattice have been defined in [9]. A nonvanishing integer value of the vortex operator σ_P on some plaquette P signals the presence of a vortex. The lattice gauge coupling β_G is related to the $3D$ continuum gauge coupling g_3^2 and controls the continuum limit $\beta_G = 4/(a g_3^2)$ ($g_3^2 \approx g_4^2\ T$). The hopping parameter β_H is related to the temperature T (with the higher temperature, symmetric side at $\beta_H < \beta_H^{\rm cross}$).

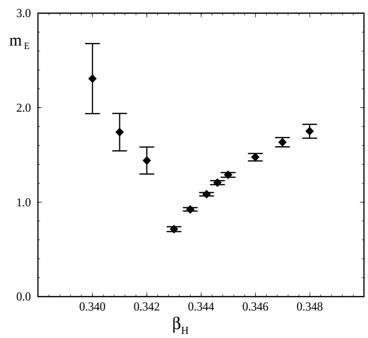
2. Vortex profile

Our vortex defect operator σ_P is constructed to localize a line-like object (in $3D$ space–time) with non-zero vorticity on the dual lattice. Within a given gauge field–Higgs configuration, a profile around that vortex "soul" would be hidden among quantum fluctuations. However, an average over all vortices in a quantum ensemble clearly reveals a structure that can be compared with a classical vortex [3, 2]. We have studied correlators of σ_P with various operators constructed on the lattice ("quantum vortex profiles").

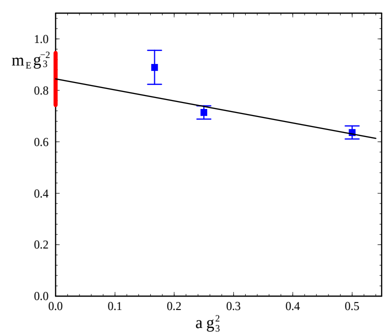
Classically, in the center of a vortex the Higgs field modulus turns to zero and the energy density becomes maximal [3, 2]. What can be expected in a thermal ensemble is, that along the vortex soul the (squared) modulus of the Higgs field and the gauge field energy density, $E_P^g = 1 - \frac{1}{2}{\rm Tr}U_P$, substantially differ from the bulk averages characterizing the corresponding homogeneous phase.[1] Indeed, in our lattice study they were found lower (or higher, respectively), with the difference growing entering deeper into the "broken phase" side of the crossover [6] (see Figure 1).

To proceed we have studied, among others, the vortex–gluon energy correlator for plaquettes P_0 and P_R located in the same plane (perpendicular

[1]Just on the "broken" side of the crossover, for instance, one would expect to find a core of "symmetric" matter inside the vortex.



(a)



(b)

Figure 2. (a) The effective mass m_E *vs.* hopping parameter β_H at $\beta_G = 16$ on the lattice 32^3; (b) Extrapolation of the mass m_E fitted at crossover to the limit $a \to 0$.

to a segment of the vortex path)

$$C_E(R) = \langle \sigma^2_{P_0}\, E^g_{P_R} \rangle\,, \tag{1}$$

as function of the distance R between the plaquettes.[2] To parametrize the vortex shape we fit the correlator data (1) by an ansatz $C^{\rm fit}_E(R) = C_E + B_E\, G(R; m_E)$ with constants C_E and B_E and an inverse penetration depth (effective mass m_E). The function $G(R; m)$ is the $3D$ scalar lattice propagator with mass $2\sinh(m/2)$ which, instead of a pure exponential, has been proposed to fit *point–point* correlators in Ref. [11].

If the quantum vortex profile should interpolate between the interior of the vortex and the asymptotic approach to the vacuum, we can only expect to describe the profile by such an ansatz for distances $R > R_{\rm min}$. The distance $R_{\rm min}$ (core size) should be fixed in physical units. Therefore we choose (in lattice units) $R_{\rm min}(\beta_G){=}\beta_G/8$ for $\beta_G{=}8,16,24$ which corresponds to $R^{\rm core}{=}aR_{\rm min}{=}(2g_3^2)^{-1}$. How successful this is to define the vortex core can be assessed studying $\chi^2/d.o.f.$ *vs.* $R_{\rm min}$ (to be reported elsewhere).

An example of the behaviour of the effective mass m_E is shown in Figure 2(a). The mass reaches its minimum at the crossover point $\beta_H^{\rm cross}$. Deeper on the symmetric side the quantum vortex profiles are squeezed compared to the classical ones due to Debye screening leading to a smaller coherence length. Approaching the crossover from this side the density of the vortices decreases thereby diminishing this effect. The extrapolation of the mass m_E (as defined at the crossover temperature) towards the continuum limit is shown in Figure 2(b).

[2] A similar method has been used to study the physical properties of Abelian monopoles in $SU(2)$ gluodynamics, Ref. [10].

3. Inter–vortex interactions and the type of the vortex medium

In the case of a superconductor, the inter–vortex interactions define the type of superconductivity. If two parallel static vortices with the same sense of vorticity attract (repel) each other, the substance is said to be a type–I (type–II) superconductor. To investigate the vortex–vortex interactions we have measured two–point functions of the vortex currents:

$$\langle|\sigma_{P_0}|\,|\sigma_{P_R}|\rangle = 2(g_{++}+g_{+-})\,, \quad \langle\sigma_{P_0}\,\sigma_{P_R}\rangle = 2(g_{++}-g_{+-})\,, \qquad (2)$$

where $g_{+\pm}(R)$ stands for contributions to the correlation functions from parallel/anti–parallel vortices piercing a plane in plaquettes P_0 and P_R. Properly normalized, the correlators $g_{+\pm}(R)$ can be interpreted as the average density of vortices (anti-vortices), relative to the bulk density, at distance R from a given vortex.

Hence the long range tail of the function g_{++} is crucial for the type of the vortex medium: in the case of attraction (repulsion) between same sign vortices g_{++} exponentially approaches unity from above (below) while g_{+-} is always attractive, independently on the type of superconductivity.

We have seen in our calculations [13] that the tail of g_{++} belongs to the attraction case (with minimal slope at the crossover). Therefore, electroweak matter in the crossover regime belongs to the type–I vortex vacuum class.

Acknowledgments

The authors are grateful to P. van Baal, H. Markum, V. Mitrjushkin, S. Olejnik and M. I. Polikarpov for useful discussions. M. Ch. feels much obliged for the kind hospitality extended to him at the Max-Planck-Institute for Physics in Munich. M. Ch. was supported by the grants INTAS-96-370, RFBR-99-01-01230 and ICFPM fellowship (INTAS-96-0457).

References

1. T. Vachaspati and M. Barriola, *Phys. Rev. Lett.* **69** (1992) 1867.
2. M. Barriola, T. Vachaspati, M. Bucher, *Phys. Rev.* **D50** (1994) 2819.
3. Y. Nambu, *Nucl. Phys.* **B130** (1977) 505.
4. N. S. Manton, *Phys. Rev.* **D28** (1983) 2019.
5. M. N. Chernodub *et al.*, *Phys. Lett.* **B434** (1998) 83.
6. M. N. Chernodub *et al.*, *Phys. Lett.* **B443** (1998) 244.
7. K. Kajantie *et al*, *Phys. Rev. Lett.* **77** (1996) 2887; M. Gürtler, E.–M. Ilgenfritz, A. Schiller, *Phys. Rev.* **D56** (1997) 3888.
8. K. Kajantie *et al.*, *Nucl. Phys.* **B458** (1996) 90; M. Gürtler *et al.*, *ibid.* **B483** (1997) 383.
9. M. N. Chernodub, F. V. Gubarev, E.–M. Ilgenfritz, *Phys. Lett.* **B424** (1998) 106.
10. S. Thurner *et al.*, *Phys. Rev.* **D** 54 (1996) 3457; M. Feurstein, H. Markum and S. Thurner, *Phys.Lett.* **B396** (1997) 203; B. L. G. Bakker, M. N. Chernodub, M. I. Polikarpov, *Phys. Rev. Lett.* **80** (1998) 30.
11. J. Engels, V. K. Mitryushkin, T. Neuhaus, *Nucl. Phys.* **B440** (1995) 555.
12. E.–M. Ilgenfritz, A. Schiller and C. Strecha, *Eur. Phys. J.* **C8** (1999) 135.
13. M. N. Chernodub, E.–M. Ilgenfritz and A. Schiller, heplat/9909001.